

Hierarchical Bayesian CMB Component Separation with the No-U-Turn Sampler

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APPENDIX A: PARAMETER RESIDUAL MAPS

In this appendix we present the residuals maps obtained for our model parameters over the three validation sets. In Fig. A1 we show the CMB amplitude residual maps. In Fig. A2 we show the synchrotron and dust amplitude residual maps. In Fig. A3 we show the residual maps for the foreground spectral parameters. In Table A1 we state the median residual values and the MAD values of the residuals for each parameter and validation set.

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Parameter	Validation Set	Med(\mathcal{R}_Q)	Med(\mathcal{R}_U)	MAD(\mathcal{R}_Q)	MAD(\mathcal{R}_U)
A_{cmb}	CP(L)	$-6 \times 10^{-3} \mu\text{K}$	$3 \times 10^{-3} \mu\text{K}$	$0.07 \mu\text{K}$	$0.06 \mu\text{K}$
A_{cmb}	CP(LC)	$-3 \times 10^{-3} \mu\text{K}$	$4 \times 10^{-4} \mu\text{K}$	$0.07 \mu\text{K}$	$0.05 \mu\text{K}$
A_{cmb}	H(LC)	$1 \times 10^{-3} \mu\text{K}$	$-6 \times 10^{-5} \mu\text{K}$	$0.05 \mu\text{K}$	$0.04 \mu\text{K}$
A_s	CP(L)	$-4 \times 10^{-4} \mu\text{K}$	$3 \times 10^{-3} \mu\text{K}$	$0.4 \mu\text{K}$	$0.4 \mu\text{K}$
A_s	CP(LC)	$-2.8 \mu\text{K}$	$0.09 \mu\text{K}$	$80.5 \mu\text{K}$	$75.2 \mu\text{K}$
A_s	H(LC)	$-0.5 \mu\text{K}$	$-0.03 \mu\text{K}$	$74.1 \mu\text{K}$	$72.8 \mu\text{K}$
A_d	CP(L)	$-8 \times 10^{-5} \mu\text{K}$	$-1 \times 10^{-4} \mu\text{K}$	$5 \times 10^{-3} \mu\text{K}$	$4 \times 10^{-3} \mu\text{K}$
A_d	CP(LC)	$-1 \times 10^{-4} \mu\text{K}$	$-7 \times 10^{-5} \mu\text{K}$	$6 \times 10^{-3} \mu\text{K}$	$5 \times 10^{-3} \mu\text{K}$
A_d	H(LC)	$2 \times 10^{-4} \mu\text{K}$	$-1 \times 10^{-5} \mu\text{K}$	$3 \times 10^{-3} \mu\text{K}$	$3 \times 10^{-3} \mu\text{K}$
β_s	CP(L)	6×10^{-3}	...	0.1	...
β_s	CP(LC)	3×10^{-3}	...	0.03	...
β_s	H(LC)	-2×10^{-3}	...	0.03	...
β_d	CP(L)	1×10^{-3}	...	0.04	...
β_d	CP(LC)	3×10^{-3}	...	0.04	...
β_d	H(LC)	6×10^{-3}	...	0.02	...
T_d	CP(L)	-0.2 K	...	1.5 K	...
T_d	CP(LC)	-0.4 K	...	1.3 K	...
T_d	H(LC)	-0.3 K	...	1.0 K	...

Table A1. In columns 3 and 4 we state the medians of the residuals for the given parameter and validation set, and in columns 5 and 6 we state the corresponding MAD values. Spectral parameters are identical in Q and U . As such, we state the median and MAD values under the Q columns only. Note that, in comparing synchrotron amplitude residuals, the residuals for the CP(L) set are calculated at a reference frequency of 40 GHz, compared to 5 GHz for the CP(LC) and H(LC) sets. This results in the seemingly low level of synchrotron amplitude residuals, given the lower level of synchrotron emission at 40 GHz.

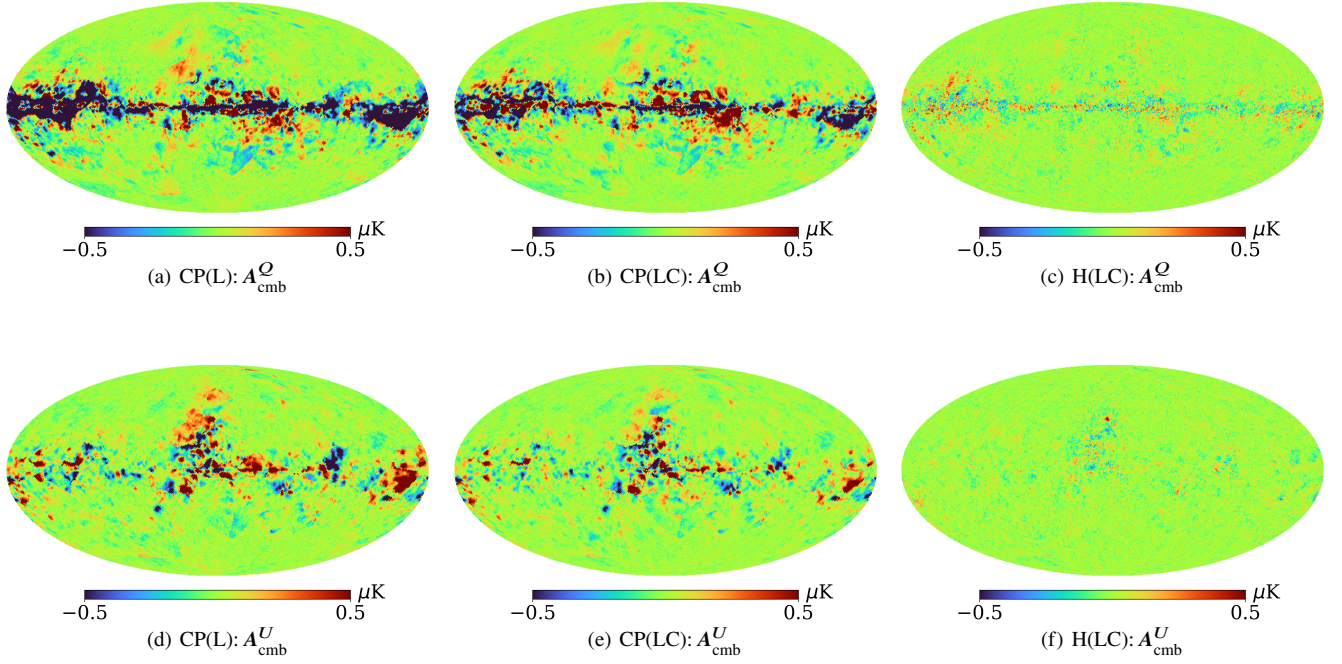


Figure A1. CMB amplitude residual maps for the three validation sets. When using the complete pooling model, strong residuals are apparent close to the Galactic plane and along bright diffuse features such as the North Polar Spur. These residuals are greatly reduced when using the hierarchical model. Large scale residuals present in the CMB amplitude maps when using the complete pooling model can ultimately translate into significant biases on recovered cosmological parameters.

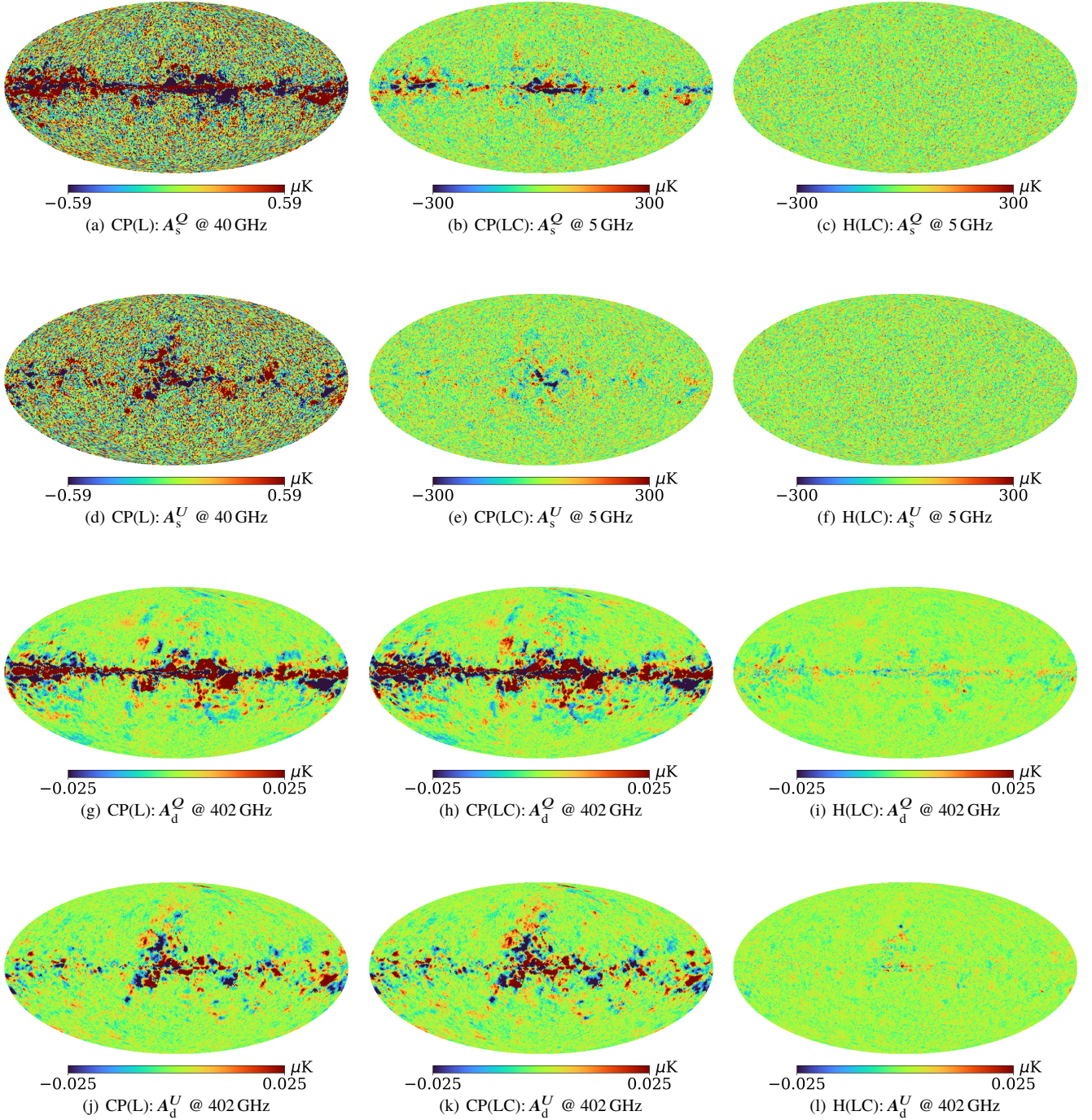


Figure A2. In panels (a) to (f) we show the synchrotron amplitude residual maps, and in panels (g) to (l) we show the dust amplitude residual maps. In displaying the synchrotron amplitude residuals for the CP(L) set, we have scaled the $\pm 300 \mu\text{K}$ axis limits used for the CP(LC) and H(LC) sets from 5 GHz to 40 GHz, using a spectral index of -3 . For both the synchrotron and dust amplitudes we can see that the residuals, particularly in regions of bright diffuse emission, are greatly reduced when using a hierarchical model. Whilst the level of these residuals is small relative to the total synchrotron and dust emission at their reference frequencies, these residuals ultimately propagate through to significant foreground residuals in the recovered CMB amplitude maps.

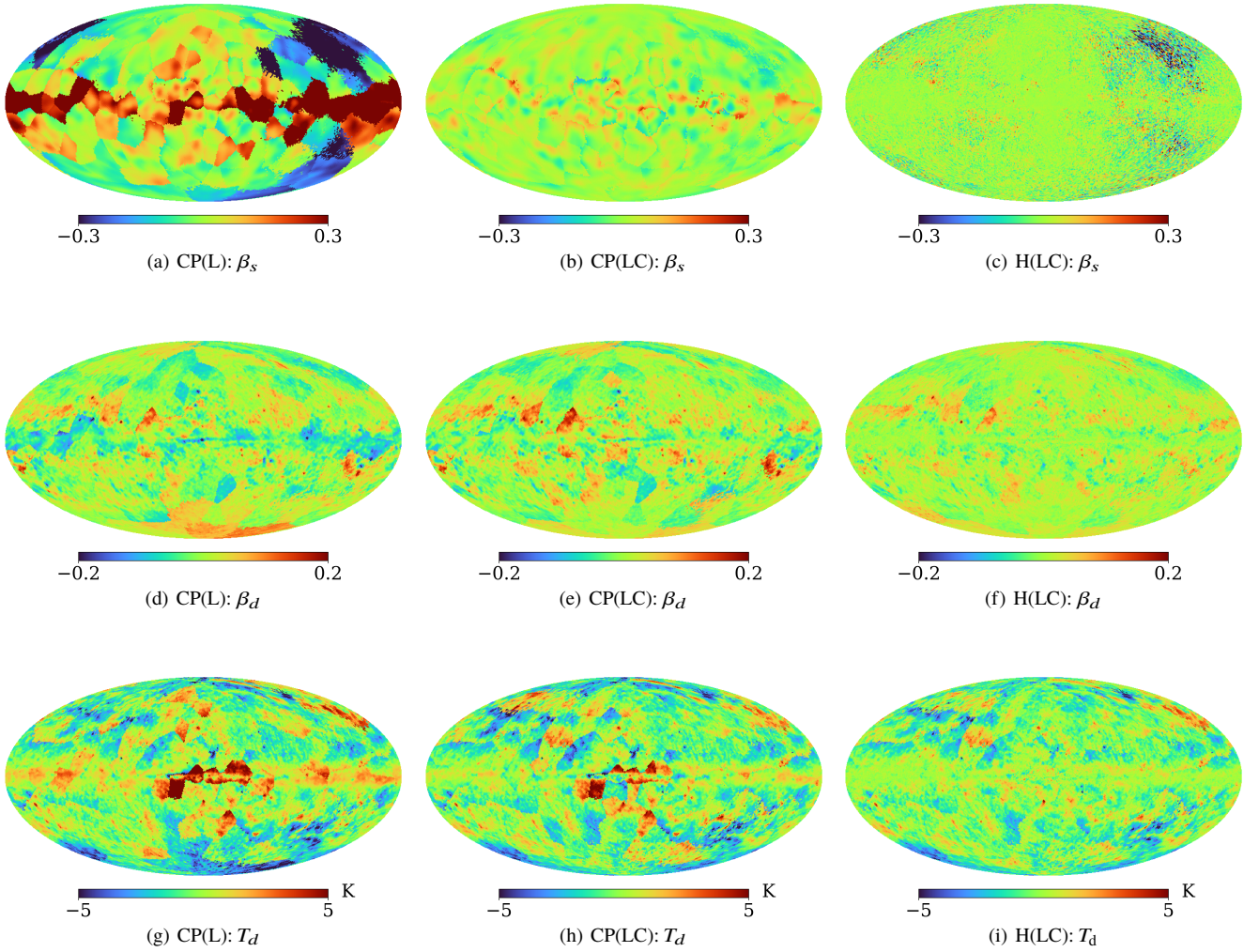


Figure A3. Spectral parameter residual maps obtained for each validation set. Panels (a) to (c): Synchrotron spectral index residual maps. Panels (d) to (f): Dust spectral index residual maps. Panels (g) to (i): Dust temperature residual maps. For the CP(L) validation set we find significant residuals for the synchrotron spectral index over much of the sky. Due to the lack of low-frequency channels below 40 GHz, *LiteBIRD* data alone is unable to properly constrain the synchrotron spectral index. In comparing the residuals for the CP(LC) and H(LC) sets, we can see that residuals are reduced for the H(LC) set in regions of bright diffuse emission. However, there are regions of low SNR away from the Galactic plane where the residuals for the H(LC) set are noticeably larger. In these regions, the low SNR means we struggle to constrain the synchrotron spectral index at the pixel-level. The fitted hyper-distributions in this case do help to reduce the propensity for the model to overreact to noise. Improved point estimates could likely be obtained here by defining larger regions in areas of low SNR. It is also worth noting that the performance of the complete pooling model is likely exaggerated by the lack of small-scale features in the simulated synchrotron spectral index map. For dust spectral parameters, we obtain lower typical residuals for the H(LC) set due to modelling real spatial variations in the parameters. However, given a maximum observed frequency of 402 GHz, we still struggle to place constraints on dust spectral parameters. Improvements can likely be achieved for the MBB model through a careful study of the choice of informative priors and possible re-parametrizations.